Optimal Countrates for Deadtime Corrections

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Abstract. The high x-ray flux available at synchrotron radiation sources can cause nonlinearities in photon-counting detectors unless deadtime corrections are employed. We compute the uncertainties associated with several common deadtime-correction formulas. At lower countrates, statistical noise dominates the error in the measured countrates; at higher countrates, the dominating factors are saturation of the response and uncertainty in the value of the deadtime parameter. In between, a range of countrates exists in which the signal-to-noise ratio can be optimized for photon-counting experiments.

Keywords: deadtime correction, counting detector, synchrotron radiation, data correction
PACS: 06.20.Dk, 07.05.Hd, 07.85.Qe, 07.85.Fv

INTRODUCTION

Deadtime corrections are used to correct nonlinearities in the output of photon-counting detectors; these nonlinearities appear when photons arrive too fast for all to be counted. The appropriate formula can extend the dynamic range of a detector beyond its linear regime, which is much more efficient than attenuating the beam to limit a detector to its linear-response range. In this paper we review several common deadtime formulas and derive the associated signal-to-noise ratios. For any measurement not limited by photon flux, these results can be used to obtain the optimal countrate for a given detector.

DEADTIME MODELS

Numerous deadtime-correction formulas have been derived for a variety of photon detector applications [1–7]. Such formulas have been derived for many cases, including whether the detector exhibits nonextended or extended behavior; whether the time structure of the x-ray beam is continuous, as a tube source, or pulsed, as a synchrotron; and whether the detector discriminates the number of simultaneously arriving photons (i.e., whether it performs pulse-height analysis). This paper analyzes three common deadtime corrections. The first, and perhaps simplest, is the nonextended deadtime model, which is applicable for a detector that is "dead" for a time τ after a given photon is *counted*. The true countrate N_T is obtained from the observed countrate N_O via the expression

$$N_T = \frac{N_O}{1-\tau N_O}. \tag{1}$$

The effective deadtime constant τ, as applied in the formulas of this paper, usually depends on a combination of intrinsic and extrinsic factors, that is, the response time of the detector and the time structure of the source, respectively.

However, most detectors are "dead" for a time τ after *each* photon *arrives* [3,5]. That is, after a photon is counted, the arrival of additional photons within τ extend the length of time that the detector is dead. This model is thus known as the extended or paralyzable deadtime model. For a detector that uses pulse-height analysis to discriminate against multiple-photon events, the appropriate formula is

CP1234, *The10th International Conference on Synchrotron Radiation Instrumentation*
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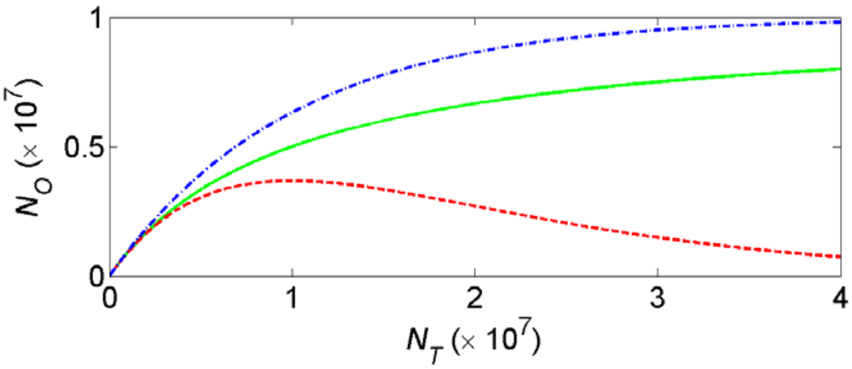

FIGURE 1. Comparison of observed and true countrates for the nonextended (solid line), extended (dashed line), and isolated (dash-dot line) deadtime models. In this example, the deadtime parameter τ = 100 ns.

$$N_O = N_T \exp(-\tau N_T)\ . \tag{2}$$

Solutions of Eq. 2 for N_T are discussed in Refs. [7,8]. This correction is typically appropriate for a scintillator detector using a single-channel analyzer to prevent harmonic contamination.

A third deadtime model is applicable to a fast detector, i.e., one whose recovery time is shorter than the pulse separation, allowing the pulses to be isolated. If pulse-height analysis is used, then Eq. 2 still applies. But without pulse-height analysis, any number of simultaneously arriving photons will be observed as a single count. This isolated deadtime model thus yields

$$N_T = \frac{-1}{\tau} \ln(1 - \tau N_O)\ . \tag{3}$$

In this case τ is simply the time between pulses from the source. This model is often applicable to an avalanche photodiode in photon-counting mode at a synchrotron source whose fill pattern has modest gaps between bunches.

A comparison of these three deadtime models is shown in Fig. 1. A value of τ = 100 ns is used to calculate these curves. Note that the maximum observed countrate N_O for the nonextended and isolated deadtime models is 1/τ. In contrast, there is a maximum value of N_O for the extended deadtime model of $(e\,\tau)^{-1}$ at N_T = 1/τ; as the true countrate is increased above 1/τ, the deadtime becomes so extended that the apparent countrate decreases, and the resulting data are not usable.

RELATIVE ERRORS OF DEADTIME MODELS

Two sources of uncertainty can be associated with the values of N_T derived from Eqs. 1, 2, or 3: the statistical error from the measurement of N_O and uncertainty in the exact value of τ. Here we use σ_X to represent the uncertainty associated with parameter X, so the two experimental uncertainties are σ_{N_O} and σ_τ, respectively. σ_{N_T} must be obtained to determine the trustworthiness of the true countrate N_T. In this section we derive the relative error of the true countrate, σ_{N_T} / N_T, by error propagation in quadrature for N_T (Eqs. 1-3) with respect to N_O and τ (see, e.g., [9]). If I_O is the observed intensity, measured as the number of photons counted in time t, then Poisson statistics give the error of the intensity as the square root of the number of counts, or $\sigma_{I_O} = \sqrt{I_O}$. The observed countrate N_O is, by definition, I_O/t, so its corresponding error is

$$\sigma_{N_O} \approx \frac{1}{t}\sqrt{I_O} = \sqrt{\frac{N_O}{t}}\ . \tag{4}$$

Eq. 4 is not strictly correct as the *true* counts, rather than the *observed* counts, should follow a Poisson distribution [1]. At low countrates, before deadtime corrections become significant, there is little difference between the observed and true countrates; Eq. 4 is safely valid in this regime. At higher countrates (beyond several percent of $1/\tau$), this approximation will overestimate the true relative error. However, as shown below, other sources of error dominate at higher countrates; the associated overestimation does not, then, affect the conclusions of our work. Via error propagation, the resulting relative error for the nonextended model (Eq. 1) is

$$\frac{\sigma_{N_T}}{N_T} = N_O^{-1/2}\left(1-\tau N_O\right)^{-1}\left(\frac{1}{t}+N_O^3\,\sigma_\tau^2\right)^{1/2} . \tag{5}$$

The first factor on the left side of this equation is large at low countrates; the second is large at high countrates. In the final factor of this equation, the first term is due to counting error, and the second is due to uncertainty in τ.

The relative error for the extended model (Eq. 2) is quite similar to Eq. 5:

$$\frac{\sigma_{N_T}}{N_T} = N_O^{-1/2}\left(1-\tau N_T\right)^{-1}\left(\frac{1}{t}+N_T^2 N_O\,\sigma_\tau^2\right)^{1/2} . \tag{6}$$

At a synchrotron source, the repetition rate of the x-ray bunches should be known, so there should be no uncertainty in τ associated with the isolated deadtime model (Eq. 3). Therefore, with $\sigma_\tau = 0$, the relative error for this model is

$$\frac{\sigma_{N_T}}{N_T} = \frac{1}{N_T\left(1-\tau N_O\right)}\sqrt{\frac{N_O}{t}} . \tag{7}$$

In Fig. 2 we plot the relative error of the three models. As with Fig. 1, we use τ = 100 ns; the count time t = 0.01 s. We present the relative error against N_O rather than N_T, as the observed countrate is the immediately accessible quantity. Three lines are plotted assuming τ is known exactly; also shown is the nonextended model with σ_τ/τ = 1%.

DISCUSSION

The relative importances of the various sources of error are clearly illustrated in Fig. 2, with several qualitative similarities between the models. At low countrates, precision is limited by statistical error. The relative error will improve with more counts, either by having a higher counting rate or by counting for longer times. In this regime, the relative error decreases as $t^{1/2}$ (Eqs. 5-7). At high countrates, large changes in N_T produce only small changes in N_O, as reflected by the flattening of the curves for the deadtime models in Fig. 1; the detector loses its sensitivity to observe small changes in the true count rate. Between the extremes, these graphs have wide minima, over which is

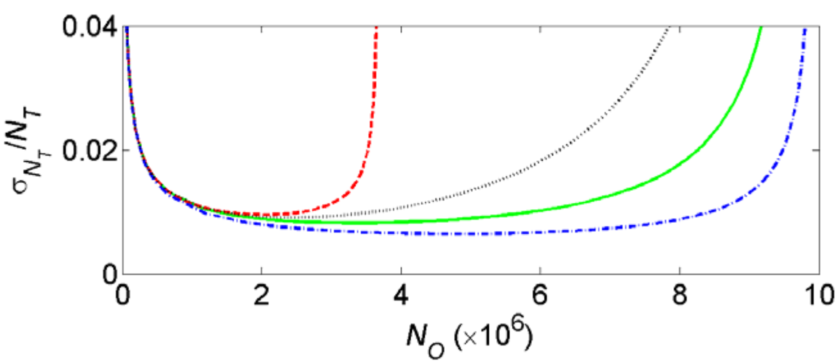

FIGURE 2. Relative error in the true countrate for the nonextended (solid line), extended (dashed line), and isolated (dash-dot line) deadtime models, with $\sigma_\tau = 0$. The dotted line is the calculation for the nonextended model including an uncertainty in the deadtime parameter of σ_τ/τ = 1%. In this example, the deadtime parameter τ = 100 ns, and the count time t = 0.01 s.

the most favorable range of countrates.

Uncertainty in the value of the deadtime parameter τ will increase errors in N_T. As demonstrated by the dotted line in Fig. 2, the effects of σ_τ are more prevalent at higher countrates and will reduce the optimal countrate N_O. Uncertainties in τ can be minimized by avoiding the high-countrate regime but cannot be eliminated; as can be seen in Eqs. 5 and 6, increasing the count time t will not reduce the effect of this source of uncertainty.

The above calculations used a value of τ = 100 ns for Fig. 2, which allowed relative errors as low as 1% for fairly short counting times. But if τ is longer, the relative error suffers accordingly. For example, in gated pump-probe experiments, $1/\tau$ for the isolated model is the repetition rate of the pump. This rate might be only on the order of a kHz. The best relative error for 1-s count times at τ = 1 ms is 7%. Longer count times become essential for improved errors.

The results we have described can be used to optimize more complicated photon-counting measurements. As an example, we present the case of determining the ratio R of two countrates A and B. This could be for a pump-probe experiment when the pump is on vs. off, or a polarization-flipping experiment when the polarization is up vs. down. The desired value is $R_T = A_T/B_T$ with uncertainty σ_{R_T}. The observed quantities are A_O and B_O with uncertainties σ_{A_O} and σ_{B_O}, respectively. For simplicity, we neglect any uncertainty in τ and assume τ is equal for both signals (in many cases, A_O and B_O are measured with the same detector). The relative error of the true ratio is

$$\frac{\sigma_{R_T}}{R_T} = \left(A_T^{-2} \left(\frac{\partial A_T}{\partial A_O} \right)^2 \sigma_{A_O}^2 + B_T^{-2} \left(\frac{\partial B_T}{\partial B_O} \right)^2 \sigma_{B_O}^2 \right)^{1/2} . \tag{8}$$

As a simple example, we consider the nonextended deadtime model. The true ratio R_T in terms of the observed countrates is $R_T = \frac{A_O(1-\tau B_O)}{B_O(1-\tau A_O)}$. Applying Eq. 1 to Eq. 8, the relative error of the ratio becomes

$$\frac{\sigma_{R_T}}{R_T} = t^{-1/2} \left(\frac{1}{A_O\left(1-\tau A_O^2\right)^2} + \frac{1}{B_O\left(1-\tau B_O^2\right)^2} \right)^{1/2} . \tag{9}$$

In a relatively low-countrate regime, the relative error will be dominated by the smaller of the two countrates, while in a higher countrate regime, it will be dominated by the greater of the two. If the ratio is near unity ($A_O \sim B_O$), then an optimal countrate can be obtained as was calculated in Fig. 2. These results can be generalized to other deadtime models or to other relative quantities, such as the difference $A-B$ or the asymmetry ratio $(A-B)/(A+B)$.

ACKNOWLEDGMENTS

The Advanced Photon Source is supported by the U. S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357.

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